LUNAR AND SOLAR BAROMETRIC TIDES IN AUSTRALIA

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ABSTRACT

The lunar semidiurnal barometric tide L_2 and the solar 24-, 12-, and 8-hr. oscillations of the surface pressure have been determined for 10 stations in Australia and on adjacent islands. At Rabaul and Moresby L_2 is considerably smaller than elsewhere in these latitudes. The characteristic annual variation of the phase—late high tide during the D season—is found at most Australian stations. But the annual amplitude minimum occurs only at half the Australian stations during this season, contrary to the behavior of L_2 over most of the globe.

1. INTRODUCTION

In order to obtain a more complete and reliable picture of the global distribution of the lunar barometric tide, hereafter called L_2, 10 determinations have been made for stations in Australia and on some nearby islands (table 1). In this region of the world the tide has heretofore only been determined for Melbourne and Wellington by Chapman and collaborators [1, 2]. With these 10 additional Australian stations lunar barometric tidal determinations are now available for 102 stations. Of these, 67 are in the Northern Hemisphere, 35 in the Southern Hemisphere, so that it will be possible to make a reasonably satisfactory spherical harmonic analysis of L_2 and its remarkable seasonal variation. Such an analysis will be carried out in a subsequent paper. Here, a report on the analysis of the lunar semidiurnal barometric oscillations in Australia will be given.

The data for this investigation, 14 yr. of surface pressure observations (from July 1951 through June 1965), were made available by the Australian Bureau of Meteorology to whom we are indebted for their assistance.

The observations were taken at 3-hr. intervals (solar time) except for 1 night hr. at which observations were not made. Under these circumstances the customary formulae for harmonic analysis, based on equidistant values, are not applicable. Instead, if we wish to determine the harmonic coefficients of the first three waves, that is for the diurnal, semidiurnal, and terdiurnal solar oscillation we have, with the mean value and the two harmonic constants for each of the three oscillations seven constants which are to be determined from only seven equations,

$$p_k=\sum_{i=0}^{3}(a_i \cos ix_k+b_i \sin ix_k),\quad b_0=0$$

where $x_k=k\cdot 45°$ and $k=0, 1 \ldots, 6$. Thus the determination of the harmonic coefficients is no longer a least squares problem. A simple computing scheme for the calculation of the a_i and b_i can easily be written down (see appendix). The Chapman-Miller [3] method was used to determine L_2 after the solar semidiurnal pressure oscillation was computed in this manner.

It is also possible to represent p_k by the daily mean, the 24-hr. and 12-hr. solar waves only, implying thereby that the deviations of the p_k from the sum of these terms are entirely noise. This is doubtless correct to a certain extent. If this view is adopted the determination of the five harmonic coefficients is to be carried out by the method of least squares. But the formulae for the computations become more complicated than with the method used here. Moreover, much of the deviation of the observed p_k from the values represented by the superposition of the 24-hr. and 12-hr. waves on the daily mean must be due to the 8-hr. wave, especially if, as here, the p_k are averages of a great number of days. Therefore, both for ease of computation and to include the 8-hr. wave in the determination, the three first solar-day waves have been computed by direct solution of the seven equations for the seven unknowns, rather than by the use of a least squares method.

The correction for linear change (Lamont correction) has been applied. It is advantageous to eliminate the effects of large aperiodic pressure variations from the determination of the lunar tide and the solar-day oscillations. Therefore, those days have been omitted when the difference between the highest and lowest pressure was equal to or larger than 5 mb. Since 14 yr. of data were available, the number of days used for the tidal determinations should give an indication how many days were undisturbed. However, the reduction of the number of days (shown in table 1) is partly also due to the fact that on many days observations were missing so that the day had to be omitted. This applies especially to Lord Howe Island. Here for the period from July 1951 through 1959 only 609 days are available with seven observations per day. Since these observations were, moreover, largely taken at other hours than those during the last 6 yr., only this latter part of the period was used for tidal determinations at this station.

At Hobart only about one-third of the total number of days can be used for the tidal determinations if days

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TABLE 1.—*Annual means of L_2*

Station	Lat.	Long.	Alt.*) (m.)	Days	l_2 (μb.)	r_2 (μb.)	λ_2
Rabaul	4.2° S	152.2° E	8	4034	47.8	3.8	73.0°
				(4071)	(53.0)	(6.4)	(65.8)
Pt. Moresby	9.4 S	147.2 E	47	4865	40.2	3.8	75.8
				(5082)	(39.6)	(5.4)	(75.2)
Darwin	12.5 S	130.9 E	29	4927	59.9	3.1	49.0
				(5100)	(51.0)	(5.3)	(41.5)
Broome	17.9 S	122.2 E	9	4386	49.5	3.6	72.2
				(4893)	(53.4)	(4.6)	(59.1)
Rockhampton	23.4 S	150.5 E	14	4516	41.7	3.8	83.0
				(5076)	(53.5)	(3.6)	(82.0)
Alice Springs	23.8 S	133.9 E	548	4409	38.2	3.9	56.4
				(5051)	(38.2)	(6.9)	(59.9)
Norfolk I.	29.0 S	167.9 E	109	4282	52.2	2.9	71.5
				(5036)	(53.6)	(4.9)	(67.6)
Kalgoorlie	30.8 S	121.4 E	361	3099	41.0	4.6	53.8
				(5015)	(31.9)	(6.5)	(49.3)
Lord Howe I.	31.5 S	159.0 E	46	1451	34.2	5.1	71.2
				(4800)	(41.7)	(6.1)	(77.8)
Hobart	42.8 S	147.5 E	55	1726	24.4	4.9	80.1
				(4452)	(30.8)	(11.8)	(70.3)
Melbourne**)	37.6 S	145.0 E	28	13013	28.5	2.3	84.0
Wellington**)	41.1 S	174.8 E	3	8903	38.9	5.6	69.3

*) Altitude of the barometer above sea level.
**) After Chapman and collaborators [1, 2].
In parentheses: Results of determinations from three values per day.

TABLE 2.—*Seasonal means of L_2*

Station	J			E			D		
	l_2 (μb.)	r_2 (μb.)	λ_2	l_2 (μb.)	r_2 (μb.)	λ_2	l_2 (μb.)	r_2 (μb.)	λ_2
Rabaul	53.5	7.0	77.3°	52.2	5.4	79.8°	39.4	5.0	56.0°
Pt. Moresby	39.2	5.2	86.8	44.8	4.4	80.4	38.8	5.6	59.5
Darwin	53.7	5.3	49.7	56.8	4.8	65.6	73.3	6.2	35.8
Broome	55.8	6.1	71.9	44.6	8.4	77.8	47.8	7.1	65.8
Rockhampton	47.9	5.7	93.0	46.5	5.2	92.9	35.5	6.5	59.9
Alice Springs	46.3	6.1	65.4	32.2	7.8	69.2	39.3	6.1	33.2
Norfolk I.	54.8	5.3	79.3	48.7	4.8	83.8	56.7	6.4	55.2
Kalgoorlie	40.0	7.4	48.2	36.6	8.7	63.9	47.5	10.3	50.3
Lord Howe I.	33.8	9.0	99.7	23.4	9.4	73.8	50.9	10.3	52.8
Hobart	12.9	8.9	32.5	29.9	7.8	103.1	38.0	10.8	77.6
Melbourne*)	25.8	3.1	100.3	26.2	4.0	73.6	35.5	4.7	79.6
Wellington*)	39.9	9.8	70.2	39.8	9.8	76.6	37.9	9.6	61.0

*) After Chapman and collaborators [1, 2].

with daily pressure ranges larger than 5 mb. are omitted. By applying a less restrictive criterion for the selection of days, namely permitting a maximum pressure difference between consecutive hours of observation of less than 4 mb., a total of about 4,500 days can be used for the determination. But with this limit many days with large aperiodic variation are included in the computations. Consequently the probable error circles for L_2 are about as large as those obtained when only the much fewer, but less disturbed days are used. The two determinations of L_2, with the more and the less restrictive criteria for the selection of days, agree within the limits indicated by the two probable error circles. Thus, there is no reason to relax the criterion for selection in the case of Hobart.

But the selection of the less disturbed days may affect the results for the solar-day oscillations, especially the 24-hr. wave because of the "curvature-effect" demonstrated by Bartels [4]: The less disturbed days occur in general during anticyclonic weather situations. Thus a slight convex curvature is superimposed on the daily pressure curve, even if the average of a great number of days is used.

Bartels [5] has described a method to determine L_2 at stations where only a very few daily observations, at regular meteorological observing hours, are available. This method was also applied here, by selecting three out of the seven daily observations. The three selected observations are always 6 hr. apart since this is the most suitable time difference for the determination of the lunar tide with Bartels' method. Days when the maximum difference between any two of the three observations was equal to or greater than 5 mb. have been excluded.

This second computation was originally undertaken only in order to have an additional, independent check for gross errors. Its results are included here in table 1 (parentheses) in order to show how the accuracy of the determinations is reduced if only three instead of seven daily observations are used.

The probable errors in the case of three daily observations have been computed by dividing the total series of 14 yr. into seven groups of 2 yr. each. The radii of the probable error circles are not as good an indication of the reliability of the determination, as they would be with a much larger number of groups. For instance, if r_2 is one-third of the amplitude the probability is as large as 1:27 that an amplitude of such magnitude may be found in random data [6] while it would be only 1:250 if there were 50 groups. This fact should be kept in mind when the results of the two computations are compared.

2. THE LUNAR TIDE

The results of the lunar tidal determinations are shown in table 1 for the annual means, and in table 2 separated according to the three seasons:

J—May, June, July, August;
E—March, April, September, October;
D—November, December, January, February.

The lunar tidal oscillation is assumed to be given in the form

$$L_2 = l_2 \sin (2\tau + \lambda_2)$$

where l_2 is the amplitude in microbars (1 μb. = 1 dyne cm.$^{-2}$), λ_2 the phase constant, τ local mean lunar time in angular measure (1 hr. = 15°). The radius of the probable error circle r_2 is given in the same units as l_2.

The geographical distribution of L_2 is shown in figure 1. Here the annual values are shown in a harmonic dial representation with the arrow centered at the location of the station and its length proportional to the amplitude. The angle between the arrow and the west-east direction is the phase constant so that the angle between the meridian and the arrow shows the local lunar time of high

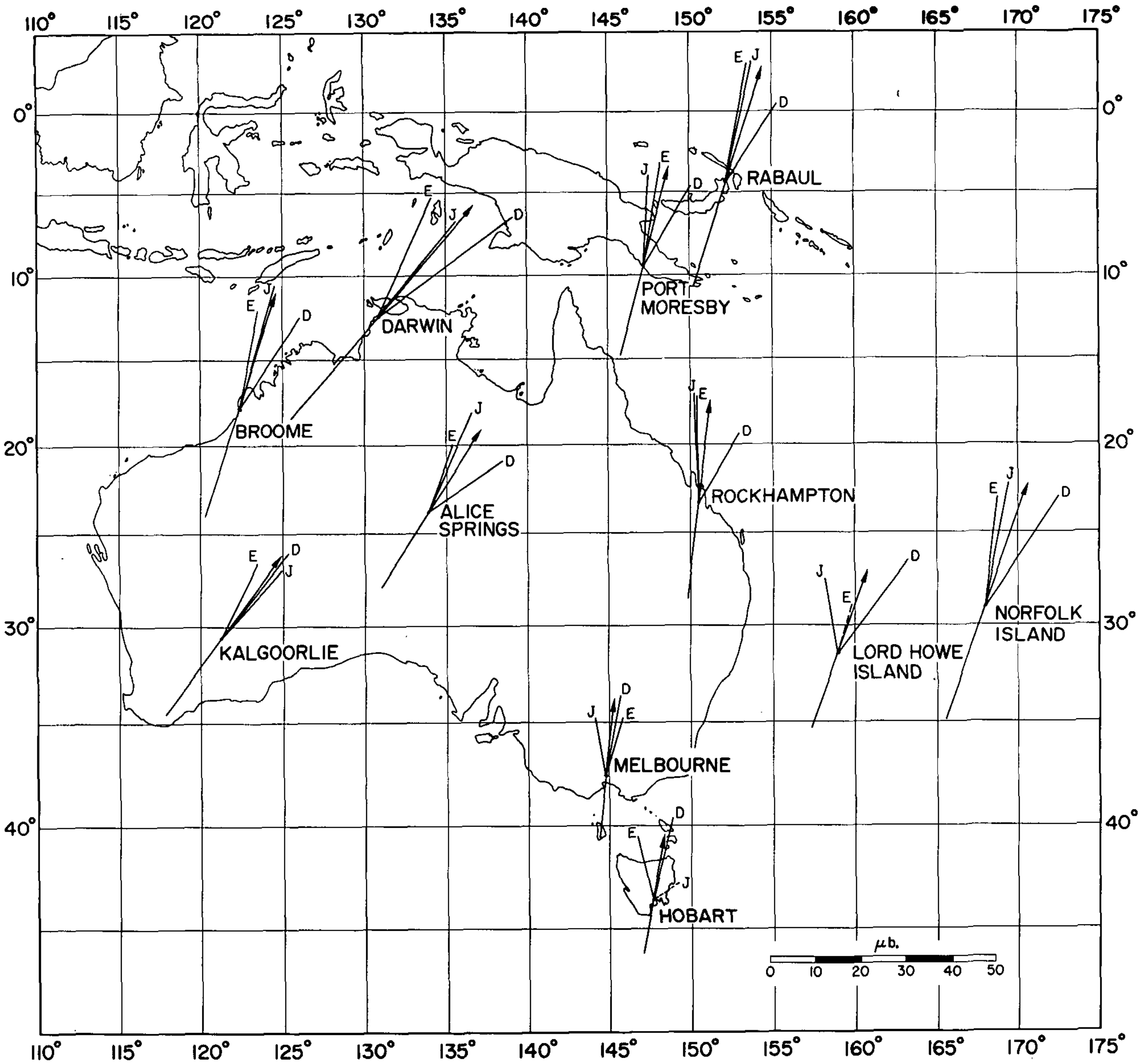

FIGURE 1.—Harmonic-dial representation of the lunar barometric tide in Australia. The lengths of the lines give the values of the amplitudes (only the upper half of each line is shown for the seasonal means); the angle between the lines and the parallels of latitude is equal to the phase constant (Melbourne after Chapman and collaborators [1]).

lunar tide as on a clock face. The seasonal values of L_2 are plotted similarly, but only the upper halves of the appropriate arrows are drawn in order to keep the figure uncluttered. At two stations, Lord Howe Island for the E months and Hobart for the J months, the probable error circle has a radius larger than one-third of the amplitude so that the determination cannot be considered satisfactory. These two cases are indicated by broken lines. The only two previous determinations of L_2 for this part of the earth, both by Chapman and collaborators, are for Melbourne [1], which is also shown in figure 1, and for Wellington [2]. Both results are included in tables 1 and 2.

The annual mean amplitudes at Rabaul and Port Moresby are small compared to those at Batavia and at most other stations in these low southerly latitudes. Only Tabora [7] in East Africa at 5°S. has a comparably low amplitude, 52 μb. Otherwise table 1 shows the expected fairly regular decrease of the amplitude with latitude with the notable exception of Norfolk Island. A remarkable feature is the late maximum tide at Darwin and to a lesser degree at Alice Springs and Kalgoorlie. Only at Haifa-Jerusalem, Kumamoto, and Naha are the phase angles smaller than at Darwin.

One of the least understood features of L_2 is its annual variation: as a rule the maximum amplitude and maximum (earliest) phase constant occur during the J months, the smallest amplitude and minimum (latest) phase constant during the D months, the change being the same in both hemispheres. Of these two variations the yearly amplitude variation is less regularly distributed over the earth than the phase variation. There are in fact numerous stations where the maximum amplitude does not occur during the J months, one of these stations being Melbourne. Table 2 shows five additional Australian stations, Darwin, Norfolk Island, Kalgoorlie, Lord Howe Island, and Hobart where the largest amplitude occurs during the D months, although at some of these stations, especially at Norfolk Island and Kalgoorlie, the probable errors are so large that the seasonal variations of the amplitude are not well established. (Similarly, the seasonal amplitude variations at some of the stations with the "normal" maximum, during the J months, are not really proven in view of the probable errors of the determinations.)

The increase of the phase constant of L_2 from the D season to the J season is a much more common phenomenon than the seasonal variation of the amplitude. The few exceptions occur where the phase constant is not well determined. This is also true for the Australian stations. Only Hobart shows an appreciably smaller phase constant in the J season than in the D season. But the probable errors, especially those for the J season, are here so large that little significance can be attributed even to this large difference of 47°. To consider the matter numerically: the radius vector of length l_2 representing L_2 in a harmonic dial makes an angle $\sin^{-1}(r_2/l_2)$ with the line from the origin tangential to the probable error circle. This angle can be taken as a measure of the uncertainty of the phase constant. For the J and D seasons at Hobart it is $\pm 44°$ and $\pm 16°$, respectively. Thus, the total uncertainty is considerably larger than the phase difference between the J and D seasons.

3. THE SOLAR-DAY OSCILLATIONS

The solar-day variations with periods of 24, 12, and 8 hr. are obtained as incidental results when the Chapman-Miller method is used for the determination of L_2. They are given in table 3. Each oscillation is assumed to be represented in the form

$$S_n = s_n \sin (nt + \sigma_n)$$

where $n=1, 2, 3$ for these three periods, and t is the local solar mean time expressed in angular measure (1 hr.=15°). The radius r_2 of the probable error circle for the semidiurnal solar oscillation is not included since it can be assumed equal to that of L_2.

Table 3 shows the well-established features of the solar-day oscillations, such as the phase shift of the 8-hr. oscillations from the J to the D season except near the Equator, and the predominance of the semidiurnal oscillation over the others. We shall here not discuss these solar-day oscillations, but only compare the solar

TABLE 3.—*Solar-day oscillations*

Station	Season	s_1	r_1	σ_1	s_2	σ_2	s_3	r_3	σ_3
Rabaul	Annual	512.2 μb.	4.3 μb.	30.7°	906.5 μb.	154.2°	77.5 μb.	2.5 μb.	21.4°
	J	535.2	8.0	33.4	815.8	154.6	75.5	5.0	30.1
	E	558.2	10.1	32.0	963.5	153.3	99.6	3.4	33.9
	D	436.8	6.9	25.1	953.7	156.7	69.7	4.4	349.0
Pt. Moresby	Annual	648.5	6.0	16.6	1003.4	161.6	82.7	2.6	40.5
	J	666.9	8.2	20.3	980.0	158.6	111.6	3.5	51.3
	E	669.1	9.2	17.0	1060.5	161.3	94.7	5.9	38.4
	D	612.5	12.3	12.1	971.0	165.0	46.7	4.7	19.4
Darwin	Annual	891.1	5.6	5.6	1145.0	157.3	51.3	3.3	350.3
	J	1035.6	8.7	11.0	1129.8	157.2	123.5	4.0	349.7
	E	937.1	9.5	3.5	1194.5	158.5	72.0	4.6	357.0
	D	708.1	11.2	0.2	1112.5	156.1	42.1	7.0	180.0
Broome	Annual	876.8	7.2	0.6	1098.6	160.9	71.0	3.7	29.9
	J	941.3	8.3	7.1	1091.8	160.6	197.9	4.2	10.6
	E	828.1	12.3	358.8	1141.3	162.4	62.8	6.0	43.0
	D	868.5	19.3	353.4	1054.9	159.4	95.9	7.9	164.9
Rockhampton	Annual	805.7	5.7	13.0	1095.5	158.3	40.6	3.0	2.5
	J	843.6	9.2	16.0	1125.1	155.2	151.6	5.6	10.2
	E	796.8	11.7	15.9	1179.6	158.8	52.6	4.7	10.1
	D	782.1	9.0	6.6	987.7	161.4	88.8	4.4	200.6
Alice Springs	Annual	1037.1	6.5	358.5	991.2	155.7	24.9	3.7	3.7
	J	848.5	9.9	1.7	1018.2	155.8	148.9	6.1	339.4
	E	1071.6	12.4	1.7	1047.3	157.9	23.3	6.9	51.1
	D	1212.5	10.8	5.0	907.2	153.1	100.3	5.0	153.8
Norfolk I.	Annual	158.4	3.6	355.9	770.0	149.0	58.2	2.2	43.6
	J	208.5	10.6	3.8	796.7	152.2	213.1	4.9	352.7
	E	197.8	5.8	358.4	783.2	149.1	87.4	4.7	33.3
	D	82.9	6.3	331.5	735.4	145.8	169.7	3.6	146.6
Kalgoorlie	Annual	1047.2	8.6	349.5	760.2	161.4	86.9	4.1	12.3
	J	694.9	16.2	348.5	736.5	159.4	226.9	4.6	1.6
	E	1067.8	14.1	350.2	811.3	163.6	88.4	6.5	11.8
	D	1455.4	19.5	349.5	728.1	161.0	96.2	7.0	159.4
Lord Howe I.	Annual	153.0	7.8	341.9	793.5	157.3	10.8	6.1	107.2
	J	232.8	19.1	26.4	776.7	156.0	175.5	9.6	18.3
	E	170.9	14.8	356.1	853.7	156.7	8.7	11.8	44.6
	D	169.6	15.5	297.9	748.7	159.0	167.8	9.6	188.9
Hobart	Annual	369.1	15.1	1.3	571.7	169.9	76.1	4.8	23.9
	J	347.2	21.3	354.8	531.6	167.4	175.3	8.6	359.3
	E	444.0	30.5	1.9	626.4	170.6	101.3	7.8	25.3
	D	328.3	30.0	8.2	564.8	171.5	80.8	8.5	140.7

semidiurnal oscillation S_2 with L_2. Only the annual means are considered since the ratios of the seasonal values are strongly influenced by the pronounced seasonal variation of L_2. Table 4 shows the amplitude ratios 100 l_2/s_2 and the phase differences $\sigma_2-\lambda_2$, the latter being a measure for the time difference between the pressure maxima reckoned from the time of transit of the tide-generating body. Chapman and Westfold [8] have compared L_2 and S_2 at 69 stations widely distributed over the globe, but they had in the region studied in this paper only two stations, Melbourne and Wellington, shown at the bottom of table 4. On the basis of their global analysis they consider ratios of 4.0 and less "specially low," ratios of 5.5 and more "specially high." Most of the stations in Australia, like Melbourne, do not have high amplitude ratios. A notable exception is Norfolk Island with its high ratio which should be compared with Wellington for which Chapman and Westfold found the highest amplitude ratio. The ratios at Moresby, Rockhampton, and Alice Springs are slightly low.

For the phase differences Chapman and Westfold designated values larger than 85° "above normal," smaller than 70° "below normal." With this definition none of the phase differences in the region of Australia are below normal. The majority are in fact above the normal values, like the difference for Wellington found by Chapman and Westfold. But Norfolk Island has a normal phase difference, its high amplitude ratio not withstanding. When considering these relations between L_2 and S_2, it must of course be remembered that both are only determined with a limited accuracy expressed by r_2, the radius of the probable error circle so that not much weight can be given to the only marginally abnormal values.

APPENDIX

DETERMINATION OF HARMONIC COEFFICIENTS IF SEVEN 3-HOURLY OBSERVATIONS PER DAY ARE GIVEN

After the Lamont correction has been applied to the observations the corrected values are

$$p_k;\ k=0, 1, \ldots, 6$$

where p_k is the pressure value at the hour 3 k. Corrections can be applied to the computed harmonic coefficients if the first observation of the day is not taken at mean local midnight.

It is assumed that the observations can be represented by a trigonometric series of the form

$$p_k=\sum_{i=0}^{3}(a_i \cos ix_k+b_i \sin ix_k) \qquad (1)$$

where $x_k=k\cdot 45°$. This expression contains seven constants since b_0 does not appear, and there are seven data points and hence seven equations are available to determine the seven harmonic coefficients, a_i, b_i.

The calculation is quite straight forward requiring the solution of a system of seven linear inhomogeneous equations. As usual in these determinations it is useful to schematize the calculation. A computing scheme is as follows:

$p_0+p_4=4\ s_0$		$p_0-p_4=4\ d_0$
$p_2+p_6=4\ s_2$		$p_2-p_6=4\ d_2$
$p_1+p_3=2\ s_1$	$p_3+p_5=2\ s_3$	$p_1+p_5=2\ s_5$
	$a_0=s_0+s_2$	
	$a_2=s_0-s_2$	
		$b_2=s_5-a_0$
$s_1-a_0=t_1$		$s_3-a_0=t_3$
$0.7071\ t_1=u_1$		$0.7071\ t_3=u_3$
	$a_1=d_0-u_3$	$b_1=d_2+u_1$
	$a_3=d_0+u_3$	$b_3=-d_2+u_1$

The factor 0.7071 is, of course, cos 45° (or sin 45°).

TABLE 4.—*Comparison of lunar and solar semidiurnal oscillations (annual means)*

Station	$100l_2/s_2$	$\sigma_2-\lambda_2$
Rabaul	5.3	81°
Moresby	4.0	86
Darwin	5.2	108
Broome	4.5	89
Rockhampton	3.8	75
Alice Springs	3.9	100
Norfolk I.	6.8	78
Kalgoorlie	5.4	108
Lord Howe I.	4.3	86
Hobart	4.3	90
Melbourne*)	4.1	83
Wellington*)	9.0	98

*) After Chapman and Westfold [8].

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